Assignment 7

Hand in: Section 7.3 no 10, Supplementary Problem no 6b, 7a, 8a, 9.

Deadline: March 16, 2018.

Section 7.3: no 10, 11, 16.

Supplementary Problems

1. Evaluate the following integrals

$$\int_0^a x^2\sqrt{a^2-x^2}dx \ ,$$

2. Prove the following formula: For any "nice" function f

$$\int_0^\pi xf(\sin x)dx = \frac{\pi}{2}\int_0^\pi f(\sin x)dx.$$

Hint: Break up the integral from 0 to $\pi/2$ and from $\pi/2$ to π.

3. Evaluate the integral

$$\int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx.$$

Hint: Use the previous problem.

4. For a continuous function f on $[-a,a]$, prove that when it satisfies

$$\int_{-a}^a fg = 0,$$

for all even, integrable functions g, it must be an odd function. Hint: Use the even-odd decomposition

$$f = f_e + f_o, \quad f_e(x) = (f(x)+f(-x))/2 \ , \quad f_o(x) = (f(x)-f(-x))/2 \ .$$

5. Evaluate the following integrals:

(a)

$$\int_0^\pi x\sin x dx \ ,$$

(b)

$$\int_0^1 \text{Arccos} x dx.$$

The inverse cosine function Arccos maps $[-1,1]$ to $[0,\pi]$.

6. Evaluate the following integrals:

   (a)
   $$\int_0^1 (1-x^2)^n dx \ ,$$

   (b)
   $$\int_0^1 x^m (\log x)^n dx, \quad m,n \in \mathbb{N}.$$

   The integrand extends to 0 at $x = 0$.

7. Study the uniform convergence for the following sequences of functions. Find the pointwise limits first.

   (a) $\left\{ \dfrac{x}{x+n} \right\}; \quad [0,\infty), \quad [0,12]$.

   (b) $\left\{ \dfrac{x^n}{1+x^n} \right\}; \quad [0,\infty), \quad [0,1], \quad [2,5]$.

8. Study the uniform convergence of the following sequence of functions by any method.

   (a) $\left\{ \dfrac{nx}{1+n^2x^2} \right\}; \quad [0,\infty)$.

   (b) $\left\{ \dfrac{\sin nx}{1+nx} \right\}; \quad [0,\infty), \quad [1,\infty)$.

9. Study the pointwise and uniform convergence of $\left\{ n^\alpha x^\beta e^{-nx} \right\}$ on $[0,\infty)$ for $\alpha, \beta > 0$.